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Emittance limitations due to collective effects for the TOTEM beams

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Summary

Observation of small-angle scattering and Landau damping of collective instabilities are partly conflicting requirements because of the small value of the transverse beam emittance. The physics requests set a maximum value for the beam emittance. The beam stability requires a minimum value. A method to find good sets of beam parameters is given.

1. Introduction

The TOTEM experiment aims mainly at measuring the total proton-proton cross section at the LHC [1]. The method used is luminosity independent. It requires simultaneous measurement of elastic scattering at low momentum transfer and total inelastic rate. It aims at an absolute error of about 1 mb. The interface between TOTEM and the LHC machine is quite intricate since a special, high-beta ($\beta^* = 1540$ m), optics is required to observe small-angle scattering. Besides, detectors, called Roman Pots (RPs), sit close to the beam axis. They are installed about 220 m from the collision point. The requested luminosity is in the range of 10^{28} cm^{-2}s^{-1}, the actual value being not critical.

The impedance of the RPs does not significantly affect the beam stability, but a potential problem is associated with the electromagnetic power deposited. The problem is not when the detector is in the IN-position. It is when it is in the OFF-position, as a cavity is then created. A mechanical way of shielding it has been found [2].

However, TOTEM beams may suffer from the resistive-wall instability induced by the collimator impedance at top energy because of the small emittance necessary for the measurements [3]. The requests for the TOTEM experiment are briefly reviewed in Section 2. Stability diagrams with maximum available octupolar strength at top energy and coherent tune shifts for the most unstable coupled-bunch mode number and head-tail mode 0 are then given and discussed in Section 3, for different sets of beam parameters. Finally, a method to find good sets of beam parameters is given in Section 4.

2. Physics requests

The measurement of the total proton-proton cross section requests an efficient detection of scattered protons with a momentum transfer squared of $-t \approx 5\times10^{-3}\ (\mathrm{GeV/c})^2$. In order to have a sufficiently high acceptance of the detector in the relevant range $-t > 5\times10^{-3}\ (\mathrm{GeV/c})^2$, the value of the minimum detectable momentum transfer squared has to be of the order of $-t_{min} \approx 1-2\times10^{-3}\ (\mathrm{GeV/c})^2$ [1]. For $-t_{min} = 10^{-3}\ (\mathrm{GeV/c})^2$ and the beam momentum p = 7000 GeV/c, the associated scattering angle is given by

$$\theta_{min} = \frac{\sqrt{-t_{min}}}{p} = \frac{\sqrt{0.001}}{7000} = 4.52\ \mu\mathrm{rad}\,. \tag{1}$$

This minimum angle θ_{min} is computed as follows. The RPs are placed tangent to the beam halo at a location where the trajectory of the scattered protons has a maximum amplitude. The bottom of the RPs has a certain thickness d as shown on Fig. 1. The bottom of the pot must be at a distance from the beam centre of at least 10σ, where σ is the rms transverse beam size at the location of the RP, in order not to intercept the protons in the secondary beam halo. This assumes that the primary collimators are set at a distance from the beam centre of $7\,\sigma$ (the secondary halo extends then up to $1.4\times7\approx10\sigma$). In principle the primary collimators could be set at a distance from the beam centre of 6σ, but we kept a safety margin of 1σ.

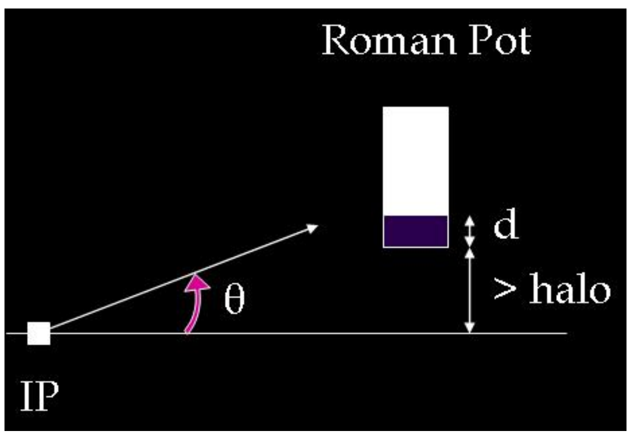


Figure 1: Rough sketch for the TOTEM experiment.

The minimum scattering angle which can be detected is then given by $\theta_{min}\,.\,L_{eff} = 10\sigma + d$, where L_{eff} = 272 m is the matrix element which transforms the angle at the interaction point (IP) in transverse displacement at the RP. For an arbitrary setting of the primary collimators, the normalised rms transverse beam emittance $\varepsilon_N = \beta_l\,\gamma\,\sigma^2/\beta$ must satisfy accordingly

$$\varepsilon_N \le \varepsilon_N^{max} = \frac{\beta_l\,\gamma}{\beta}\left(\frac{L_{eff}\,\vartheta_{min} - d}{h\,n}\right)^2, \tag{2}$$

where β = 48 m is the betatron function at the RP, β_l and γ the relativistic velocity and mass factors, $d \approx 0.5$ mm, $h = 1.4$ the gemetrical factor of the secondary halo (if the gap of the primary collimators is G = n σ, the secondary halo extends up to h n σ) [4]. A plot of ε_N^{max} vs. the gap of the collimators is represented in Fig. 2.

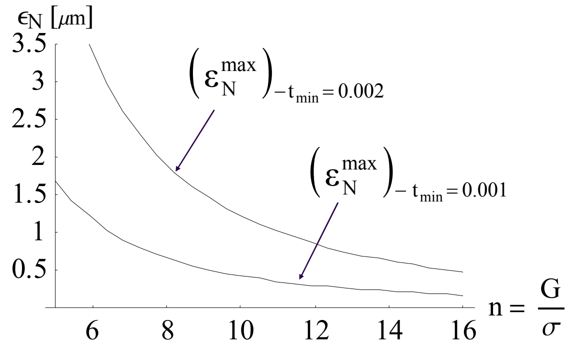


Figure 2: Plot of ε_N^{max} (due to physics requests) vs. the gap of the collimators (in beam σ). The points above the curve are forbidden.

3. Stability diagrams and coherent tune shifts

The different scenarios which have been studied first are summarised in Table 1. The gap G_0 of the collimators corresponds to the one for the nominal LHC beam [3], i.e. it is equal to 6σ of the beam with a normalised rms transverse emittance ε_N = 3.75 μm. The gap is already very small in "normal" operation, and needs to be further reduced for the TOTEM experiment in order to protect the RPs.

Table 1: Different sets of beam parameters studied.

Number of protons per bunch N_b	3×10^{10}
Number of equi-spaced bunches M	43 or 156
Normalised rms transverse emittance ε_N	1 μm
Gap of the collimators G (G_0 is the gap for the nominal LHC beam [3])	G_0 × (1, ½, ¼)

Stability diagrams corresponding to a quasi-parabolic distribution function [5] are plotted in Fig. 3 for the case $N_b = 3\times10^{10}$ p/b, $G = G_0$, ε_N = 1 μm, and ε_N = 1.2 μm. The beam is stable if the coherent tune shift for the most unstable coupled-bunch mode number and head-tail mode 0 lie inside the stable region (below the curve). It is unstable otherwise. The coherent tune shifts have been computed using Sacherer's formalim [6], taking into account the inductive-bypass effect for the resistive-wall impedance of the collimators [7].

Note that the two stability diagrams correspond to positive (a > 0) or negative (a < 0) detuning according to the sign of the octupole current, and that these curves scale linearly with the transverse beam emittance. Furthermore, the coordinates of the dot which represents the coherent tune shift, varies linearly with the number of protons per bunch. The real part of the coherent tune shift (which is related to the imaginary part of the coupling impedance) scales with $\sim G^{-3}$, whereas the instability rise-time (which is related to the real part of the coupling impedance) scales with ~G.

The results are that all the cases of Table 1 are unstable, i.e. the coherent tune shifts for the most unstable coupled-bunch mode number and head-tail mode 0 lie outside the stable region, with instability rise-times between ~10 and 300 s (see Table 2). This is due to the "almost single-bunch" effect of the inductive bypass in the resistive-wall impedance from the collimators [3]. These results could have been in fact anticipated from the results for the nominal LHC beam, where $N_b = 1.15\times10^{11}$ p/b and $\varepsilon_N = 3.75$ μm, and which is unstable [3].

Table 2: Instability rise-time for the different scenarios.

Rise-time [s]	$G = G_0 \times 1$	$G = G_0 \times ½$	$G = G_0 \times ¼$
M = 43	268	129	64
M = 156	43	19	9

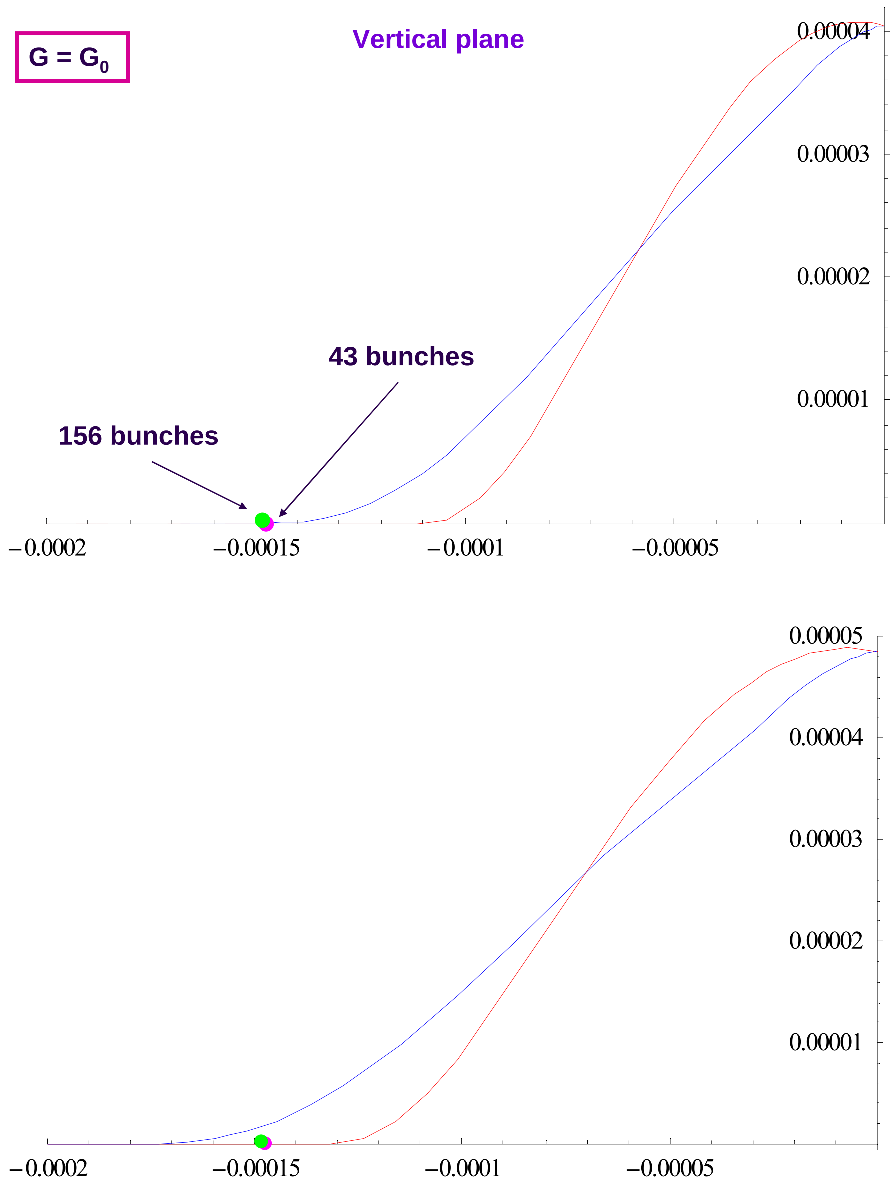


Figure 3: Stability diagrams with maximum available octupolar strength and coherent tune shifts for the most unstable coupled-bunch mode number and head-tail mode 0, and for the case $N_b = 3\times10^{10}$ p/b, $G = G_0$, $\varepsilon_N = 1$ μm (upper ⇒ unstable), and $\varepsilon_N = 1.2$ μm (lower ⇒ stable). The horizontal and vertical axes are the real and (minus) imaginary parts of the tune shift respectively.

The TOTEM beams of Table 1 have the same brightness as the nominal LHC beam, as both intensity and emitance are divided by ~4. Therefore, the real part of the coherent tune shift is ~4 times smaller, but so is the stability diagram. Eventually the beam stability is almost the same, except that now the instability rise-times are much longer (due to the "almost single-bunch" effect of the inductive bypass).

Compromises are proposed in Table 3, taking 10% safety margin for the coherent tune shift to be inside the stable region.

Table 3: Possible compromises.

$G = G_0 \times 1$	$G = G_0 \times ½$
$N_b = 3\times10^{10}$ p/b $\varepsilon_N = 1.2$ μm $\Rightarrow$ G at 10.6 σ	$N_b = 3\times10^{10}$ p/b $\varepsilon_N = 8.8$ μm $\Rightarrow$ G at 2 σ
$N_b = 2\times10^{10}$ p/b $\varepsilon_N = 0.8$ μm $\Rightarrow$ G at 13 σ	$N_b = 2\times10^{10}$ p/b $\varepsilon_N = 5.9$ μm $\Rightarrow$ G at 2.4 σ
$N_b = 1\times10^{10}$ p/b $\varepsilon_N = 0.4$ μm $\Rightarrow$ G at 18.4 σ	$N_b = 1\times10^{10}$ p/b $\varepsilon_N = 2.9$ μm $\Rightarrow$ G at 3.4 σ

Starting with the good set of beam parameters (for beam stability considerations) $N_{b0} = 3\times10^{10}$ p/b, $\varepsilon_{N0} = 1.2$ μm and $G_0 = 10.6\ \sigma_0$, it can be deduced that the following condition can be used to find another good set of beam parameters,

$$\frac{N_b}{\varepsilon_N\, G^3} \le \frac{N_{b0}}{\varepsilon_{N0}\, G_0^3}\,. \tag{3}$$

It is extremely attractive to consider cases where the beam brightness is smaller than the nominal one in order to reduce the distance of the RP to the beam. For instance for the case $N_b = 3\times10^{10}$ p/b and $\varepsilon_N = 1.2$ μm, the collimator gap set at the nominal value G_0 corresponds to 10.6σ, which is too large as 7σ is sufficient. If the beam emittance is increased to 1.97 μm, Eq. (3) is fulfilled with $G = 7\ \sigma$. In this case $-t_{min} = 1.7\times10^{-3}\ (\mathrm{GeV/c})^2$, which is satisfactory since $-t_{min}$ has to be of the order of $1-2\times10^{-3}\ (\mathrm{GeV/c})^2$ (see Section 2). It is possible to reduce further $-t_{min}$ by decreasing both the emittance and the bunch intensity as discussed below.

4. Discussion and conclusions

There is a trade-off between small-angle scattering observation and beam stabilisation by Landau damping. On the one hand, observation of small-angle scattering sets a maximum value for the transverse beam emittance, i.e. Eq. (2) has to be fulfilled. On the other hand, beam stability requires a minimum value for the transverse beam emittance, i.e. Eq. (3) has to be satisfied. The set of beam parameters ($N_{b0} = 3\times10^{10}$ p/b, $\varepsilon_{N0} = 1.2$ μm and $G_0 = n_0\ \sigma_0$, with $n_0 = 10.6$) satisfies both conditions. This is therefore, a good set of beam parameters for the TOTEM experiment. The other good sets of beam parameters can be found as follows. Noting that the gap G is related to the emittance through

$$G = G_0 \frac{n}{n_0} \sqrt{\frac{\varepsilon_N}{\varepsilon_{N0}}}\,, \qquad (4)$$

the conditions of Eqs. (2) and (3) can be put together in the following equation

$$\varepsilon_N^{min} = \varepsilon_{N0} \left(\frac{N_b}{N_{b0}} \right)^{2/5} \left(\frac{n_0}{n} \right)^{6/5} \le \varepsilon_N \le \varepsilon_N^{max} = \frac{\beta_1 \gamma}{\beta} \left(\frac{L_{eff}\, \vartheta_{min} - d}{h\, n} \right)^2 . \qquad (5)$$

These conditions are plotted in Figs. 4 and 5 for two different numbers of protons per bunch, for $-t_{min} = 10^{-3}\ (\mathrm{GeV/c})^2$ and $-t_{min} = 2\times10^{-3}\ (\mathrm{GeV/c})^2$ respectively.

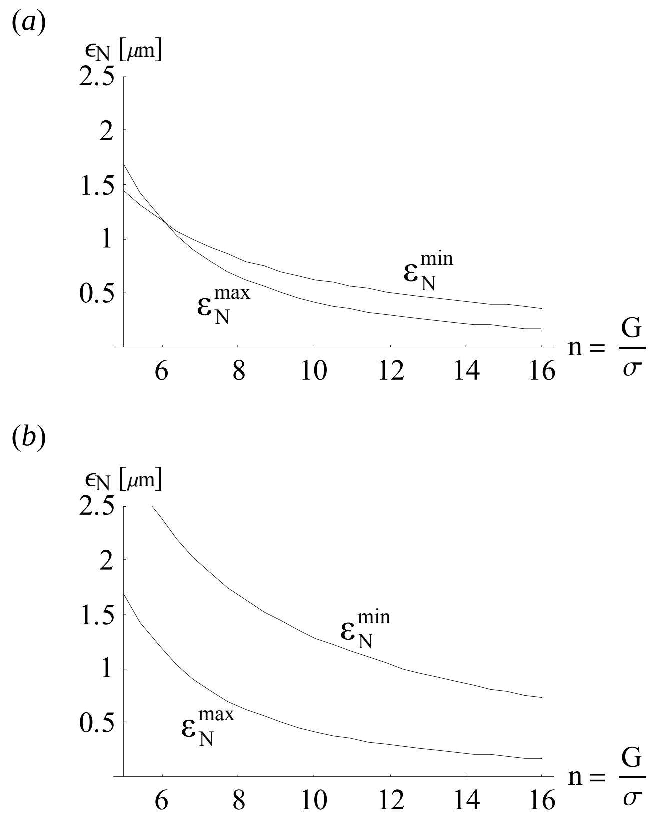
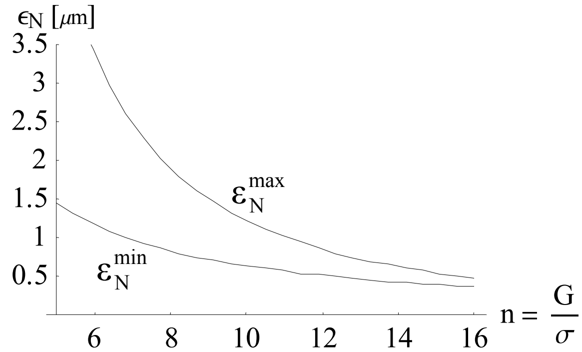


Figure 4: Minimum (due to beam stability condition) and maximum (due to physics requests) normalised rms transverse beam emittances vs. the gap of the collimators (in beam σ) for $-t_{min} = 10^{-3}\ (\mathrm{GeV/c})^2$ and (*a*) $N_b = 5\times10^9$ p/b, (*b*) $N_b = 3\times10^{10}$ p/b.

If the RPs have to be protected against the secondary halo, it is important to know the value of the emittance associated with a collimator gap equal to 7σ as it provides the smallest value of θ_{min}. This corresponds to the points associated with n = 7 on Figs. 4 and 5. These points are plotted on Fig. 6 as a function of the number of protons per bunch.

(*a*)



(*b*)

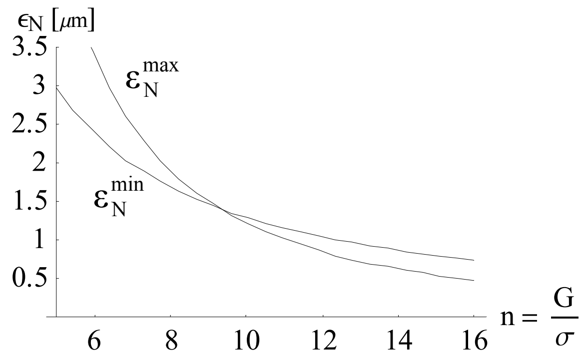


Figure 5: Minimum (due to beam stability condition) and maximum (due to physics requests) normalised rms transverse beam emittances vs. the gap of the collimators (in beam σ) for $-t_{min} = 2\times10^{-3}\ (\mathrm{GeV/c})^2$ and (*a*) $N_b = 5\times10^9$ p/b, (*b*) $N_b = 3\times10^{10}$ p/b.

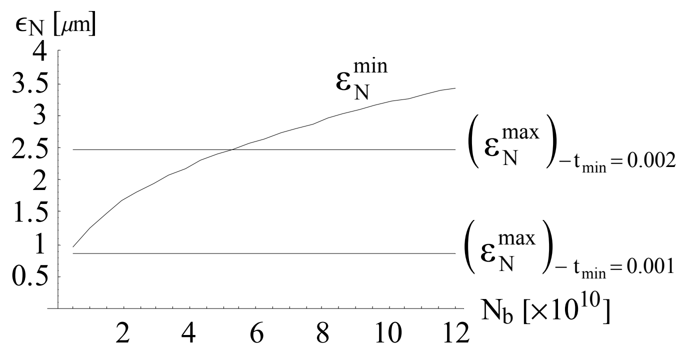


Figure 6: Minimum (due to beam stability condition) and maximum (due to physics requests) normalised rms transverse beam emittances vs. the number of protons per bunch for the gap of the collimators G = 7σ.

In conclusion, the smallest value of θ_{min} achievable is obtained for $N_b = 5\times10^9$ p/b, $\varepsilon_N = 0.96$ μm and G = 7σ, and is given by

$$\vartheta_{min}^{0} = \frac{\left(h\, n \sqrt{\frac{\beta\, \varepsilon_N}{\beta_1\, \gamma}} + d \right)}{L_{eff}} = 4.67\ \mu\text{rad}\,, \tag{6}$$

which yields $-t_{min} = 10^{-3}\ (\text{GeV}/c)^2$.

Finally, it should be noticed that the beam stability from Landau damping has been evaluated up to this point for the case of a quasi-parabolic distribution function, which extends up to 3.2σ in transverse space [5]. This distribution function underestimates the beam stability if the transverse beam profile extends up to 6σ, as it is foreseen to be the case in the LHC at top energy with the nominal collimator settings. The Gaussian distribution extends to infinity in transverse space and thus overestimates the beam stability. In Ref. [8], the beam stability has been analyzed for the distribution function consistent with the collimator settings at top energy, i.e. extending up to 6σ in transverse space. The result is that a factor of ~2 is gained for the real part of the coherent tune shift compared to the case with the quasi-parabolic distribution function. This gain can be used either to reduce the value of the emittance by the same factor 2, the corresponding momentum transfer squared then becomes $-t_{min} = 0.72\times10^{-3}\ (\text{GeV}/c)^2$, or to increase the beam intensity. For $\varepsilon_N = 1\ \mu$m, the number of protons per bunch can be increased to $N_b = 1.04\times10^{10}$ p/b. The corresponding momentum transfer squared is $-t_{min} = 1.1\times10^{-3}\ (\text{GeV}/c)^2$.

The case of a distribution extending up to $6\,\sigma$ but with more populated tails than the Gaussian distribution has also been considered (as this may be the case in reality in proton machines, where several diffusive mechanisms can take place, in particular during beam acceleration) and revealed that a factor of ~4 is gained in this case for the real part of the coherent tune shift compared to the case with the quasi-parabolic distribution function. This gain can be used either to reduce the value of the emittance by the same factor 4, the corresponding momentum transfer squared then becomes $-t_{min} = 0.52\times10^{-3}\ (\text{GeV}/c)^2$ (i.e. close to the Coulomb region [9]), or to increase the beam intensity. For $\varepsilon_N = 1\ \mu$m, the number of protons per bunch can be increased to $N_b = 2.08 \times 10^{10}$ p/b (the corresponding momentum transfer squared is the same as before, i.e. $-t_{min} = 1.1\times10^{-3}\ (\text{GeV}/c)^2$).

However, the new stability diagrams of Ref. [8] should be used with care for beam stability predictions, as the presence or not of the high-amplitude tails in the distribution can substantially affect the amount of Landau damping. This is why the analysis has been made in this note considering the "conservative" quasi-parabolic distribution function, which does not take into account the beneficial effect on Landau damping of the particles with transverse amplitudes larger than $3.2\,\sigma$. Furthermore, reducing the normalised rms transverse beam emittance down to ~0.24 μm (for the case where $-t_{min} = 0.52\times10^{-3}\ (\text{GeV}/c)^2$) is far from being easy.

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